[1] $\frac{\tan x - \sin x \cos x}{1+\cos x} = \frac{\frac{\sin x}{\cos x} - \sin x \cos x}{1+\cos x} \cdot \frac{\cos x}{\cos x}$

$= \frac{\sin x - \sin x \cos^2 x}{(1+\cos x)\cos x}$

$= \frac{\sin x (1-\cos^2 x)}{(1+\cos x)\cos x}$

$= \frac{\sin x \cancel{(1+\cos x)}(1-\cos x)}{\cancel{(1+\cos x)}\cos x}$

$= \frac{\sin x - \sin x \cos x}{\cos x}$

$= \tan x - \sin x$ QED

[2] $(\sec y + \cot y)^2 - (\csc y + \tan y)^2$

$= \sec^2 y + 2\sec y \cot y + \cot^2 y - (\csc^2 y + 2\csc y \tan y + \tan^2 y)$

$= \sec^2 y - \tan^2 y + \cot^2 y - \csc^2 y + 2\frac{1}{\cancel{\cos y}}\frac{\cancel{\cos y}}{\sin y} - 2\frac{1}{\cancel{\sin y}}\frac{\cancel{\sin y}}{\cos y}$

$= 1 - 1 + \frac{2}{\sin y} - \frac{2}{\cos y}$

$= 2\csc y - 2\sec y$

[3] $\csc t - \csc(\frac{\pi}{2}-t)\csc(-t) + \csc(\frac{\pi}{2}-t)$

$= \csc t - \sec t(-\csc t) + \sec t$

$= \csc t + \sec t \csc t + \sec t$

$\tan t + \sec(-t) + \tan(\frac{\pi}{2}-t) + \sec(\frac{\pi}{2}-t)$

$= \tan t + \sec t + \cot t + \csc t$

$= \csc t + \frac{\sin t}{\cos t} + \frac{\cos t}{\sin t} + \sec t$

$= \csc t + \frac{\sin^2 t + \cos^2 t}{\cos t \sin t} + \sec t$

$= \csc t + \frac{1}{\cos t \sin t} + \sec t$

$= \csc t + \sec t \csc t + \sec t$

[4] $\tan^4 A + \sec^2 A = (\sec^2 A - 1)^2 + \sec^2 A$

$= \sec^4 A - 2\sec^2 A + 1 + \sec^2 A$

$= \sec^4 A - \sec^2 A + 1 = \sec^4 A - (\sec^2 A - 1)$

$= \sec^4 A - \tan^2 A$

[5] $\dfrac{1}{\sec^2 B + \csc^2 B} = \dfrac{1}{\frac{1}{\cos^2 B} + \frac{1}{\sin^2 B}} \cdot \dfrac{\cos^2 B \sin^2 B}{\cos^2 B \sin^2 B}$

$= \dfrac{\cos^2 B \sin^2 B}{\sin^2 B + \cos^2 B}$

$= \dfrac{\cos^2 B (1 - \cos^2 B)}{1}$

$= \cos^2 B - \cos^4 B$

[6] $\cot^2 C + \sec^2 C \csc^2 C = \csc^2 C - 1 + (\tan^2 C + 1)\csc^2 C$

$= \csc^2 C - 1 + \tan^2 C \csc^2 C + \csc^2 C$

$= \dfrac{\sin^2 C}{\cos^2 C} \dfrac{1}{\sin^2 C} - 1 + 2\csc^2 C$

$= \dfrac{1}{\cos^2 C} - 1 + 2\csc^2 C$

$= \sec^2 C - 1 + 2\csc^2 C$

$= \tan^2 C + 2\csc^2 C$

[7] $\sec\alpha \csc\alpha - \csc\alpha = \dfrac{1}{\cos\alpha} \dfrac{1}{\sin\alpha} - \dfrac{1}{\sin\alpha}$

$= \dfrac{1 - \cos\alpha}{\cos\alpha \sin\alpha}$

$\dfrac{\tan\alpha}{1+\cos\alpha} = \dfrac{\frac{\sin\alpha}{\cos\alpha}}{1+\cos\alpha} \cdot \dfrac{\cos\alpha}{\cos\alpha}$

$= \dfrac{\sin\alpha}{\cos\alpha(1+\cos\alpha)} \cdot \dfrac{1-\cos\alpha}{1-\cos\alpha}$

$= \dfrac{\sin\alpha(1-\cos\alpha)}{\cos\alpha(1-\cos^2\alpha)}$

$= \dfrac{\sin\alpha(1-\cos\alpha)}{\cos\alpha \sin^2\alpha}$

$= \dfrac{1-\cos\alpha}{\cos\alpha \sin\alpha}$

[8] $\frac{1}{1+\sin(\frac{\pi}{2}-\beta)} - \frac{1}{1-\sec(-\beta)} = \frac{1}{1+\cos\beta} - \frac{1}{1-\sec\beta}$

$= \frac{1}{1+\cos\beta} - \frac{1}{1-\frac{1}{\cos\beta}} \frac{\cos\beta}{\cos\beta}$

$= \frac{1}{1+\cos\beta} - \frac{\cos\beta}{\cos\beta - 1}$

$= \frac{1}{1+\cos\beta} + \frac{\cos\beta}{1-\cos\beta}$

$= \frac{1-\cancel{\cos\beta}+\cancel{\cos\beta}+\cos^2\beta}{1-\cos^2\beta}$

$= \frac{1+\cos^2\beta}{\sin^2\beta}$

$= \csc^2\beta + \cot^2\beta$

[9] $(\sec\lambda - \csc\lambda\sec\lambda)(\sec\lambda + \csc\lambda\sec\lambda)$

$= \sec\lambda(1-\csc\lambda)\sec\lambda(1+\csc\lambda)$

$= \sec^2\lambda(1-\csc^2\lambda)$

$= \sec^2\lambda(-\cot^2\lambda)$

$= \frac{1}{\cancel{\cos^2\lambda}} \cdot -\frac{\cancel{\cos^2\lambda}}{\sin^2\lambda}$

$= -\frac{1}{\sin^2\lambda}$

$= -\csc^2\lambda$

[10] $\frac{\cot^4\theta - \csc^2\theta}{\csc^2\theta} = \frac{(\cot^2\theta - \csc^2\theta)(\cot^2\theta + \csc^2\theta)}{\csc^2\theta}$

$= \frac{-(\csc^2\theta - 1 + \csc^2\theta)}{\csc^2\theta}$

$= \frac{1-2\csc^2\theta}{\csc^2\theta}$

$= \sin^2\theta - 2$