

$$\begin{aligned}
[1] \quad \frac{\tan x - \sin x \cos x}{1 + \cos x} &= \frac{\frac{\sin x}{\cos x} - \sin x \cos x}{1 + \cos x} \cdot \frac{\cos x}{\cos x} \\
&= \frac{\sin x - \sin x \cos^2 x}{(1 + \cos x) \cos x} \\
&= \frac{\sin x (1 - \cos^2 x)}{(1 + \cos x) \cos x} \\
&= \frac{\sin x \cancel{(1 + \cos x)} (1 - \cos x)}{\cancel{(1 + \cos x)} \cos x} \\
&= \frac{\sin x - \sin x \cos x}{\cos x} \\
&= \tan x - \sin x \quad \text{QED}
\end{aligned}$$

$$\begin{aligned}
[2] \quad (\sec y + \cot y)^2 - (\csc y + \tan y)^2 \\
&= \sec^2 y + 2 \sec y \cot y + \cot^2 y - (\csc^2 y + 2 \csc y \tan y + \tan^2 y) \\
&= \sec^2 y - \tan^2 y + \cot^2 y - \csc^2 y + 2 \frac{1}{\cos y} \frac{\cos y}{\sin y} - 2 \frac{1}{\sin y} \frac{\sin y}{\cos y} \\
&= 1 - 1 + \frac{2}{\sin y} - \frac{2}{\cos y} \\
&= 2 \csc y - 2 \sec y
\end{aligned}$$

$$\begin{aligned}
[3] \quad \csc t - \csc\left(\frac{\pi}{2} - t\right) \csc(-t) + \csc\left(\frac{\pi}{2} - t\right) \\
&= \csc t - \sec t (-\csc t) + \sec t \\
&= \csc t + \sec t \csc t + \sec t \\
&\quad \tan t + \sec(-t) + \tan\left(\frac{\pi}{2} - t\right) + \sec\left(\frac{\pi}{2} - t\right) \\
&= \tan t + \sec t + \cot t + \csc t \\
&= \csc t + \frac{\sin t}{\cos t} + \frac{\cos t}{\sin t} + \sec t \\
&= \csc t + \frac{\sin^2 t + \cos^2 t}{\cos t \sin t} + \sec t \\
&= \csc t + \frac{1}{\cos t \sin t} + \sec t \\
&= \csc t + \sec t \csc t + \sec t
\end{aligned}$$

$$\begin{aligned}
 [4] \quad \tan^4 A + \sec^2 A &= (\sec^2 A - 1)^2 + \sec^2 A \\
 &= \sec^4 A - 2\sec^2 A + 1 + \sec^2 A \\
 &= \sec^4 A - \sec^2 A + 1 = \sec^4 A - (\sec^2 A - 1) \\
 &= \sec^4 A - \tan^2 A
 \end{aligned}$$

$$\begin{aligned}
 [5] \quad \frac{1}{\sec^2 B + \csc^2 B} &= \frac{1}{\frac{1}{\cos^2 B} + \frac{1}{\sin^2 B}} \cdot \frac{\cos^2 B \sin^2 B}{\cos^2 B \sin^2 B} \\
 &= \frac{\cos^2 B \sin^2 B}{\sin^2 B + \cos^2 B} \\
 &= \frac{\cos^2 B (1 - \cos^2 B)}{1} \\
 &= \cos^2 B - \cos^4 B
 \end{aligned}$$

$$\begin{aligned}
 [6] \quad \cot^2 C + \sec^2 C \csc^2 C &= \csc^2 C - 1 + (\tan^2 C + 1) \csc^2 C \\
 &= \csc^2 C - 1 + \tan^2 C \csc^2 C + \csc^2 C \\
 &= \frac{\sin^2 C}{\cos^2 C} \frac{1}{\sin^2 C} - 1 + 2 \csc^2 C \\
 &= \frac{1}{\cos^2 C} - 1 + 2 \csc^2 C \\
 &= \sec^2 C - 1 + 2 \csc^2 C \\
 &= \tan^2 C + 2 \csc^2 C
 \end{aligned}$$

$$\begin{aligned}
 [7] \quad \sec \alpha \csc \alpha - \csc \alpha &= \frac{1}{\cos \alpha} \frac{1}{\sin \alpha} - \frac{1}{\sin \alpha} \\
 &= \frac{1 - \cos \alpha}{\cos \alpha \sin \alpha}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\tan \alpha}{1 + \cos \alpha} &= \frac{\frac{\sin \alpha}{\cos \alpha}}{1 + \cos \alpha} \cdot \frac{\cos \alpha}{\cos \alpha} \\
 &= \frac{\sin \alpha}{\cos \alpha (1 + \cos \alpha)} \cdot \frac{1 - \cos \alpha}{1 - \cos \alpha} \\
 &= \frac{\sin \alpha (1 - \cos \alpha)}{\cos \alpha (1 - \cos^2 \alpha)} \\
 &= \frac{\sin \alpha (1 - \cos \alpha)}{\cos \alpha \sin^2 \alpha} \\
 &= \frac{1 - \cos \alpha}{\cos \alpha \sin \alpha}
 \end{aligned}$$

$$\begin{aligned}
[8] \quad \frac{1}{1+\sin\left(\frac{\pi}{2}-\beta\right)} - \frac{1}{1-\sec\beta} &= \frac{1}{1+\cos\beta} - \frac{1}{1-\sec\beta} \\
&= \frac{1}{1+\cos\beta} - \frac{1}{1-\frac{1}{\cos\beta}} \cdot \frac{\cos\beta}{\cos\beta} \\
&= \frac{1}{1+\cos\beta} - \frac{\cos\beta}{\cos\beta-1} \\
&= \frac{1}{1+\cos\beta} + \frac{\cos\beta}{1-\cos\beta} \\
&= \frac{1-\cos\beta + \cos\beta + \cos^2\beta}{1-\cos^2\beta} \\
&= \frac{1+\cos^2\beta}{\sin^2\beta} \\
&= \csc^2\beta + \cot^2\beta
\end{aligned}$$

$$\begin{aligned}
[9] \quad (\sec\lambda - \csc\lambda \sec\lambda)(\sec\lambda + \csc\lambda \sec\lambda) \\
&= \sec\lambda(1 - \csc\lambda)\sec\lambda(1 + \csc\lambda) \\
&= \sec^2\lambda(1 - \csc^2\lambda) \\
&= \sec^2\lambda(-\cot^2\lambda) \\
&= \frac{1}{\cos^2\lambda} \cdot -\frac{\cos^2\lambda}{\sin^2\lambda} \\
&= -\frac{1}{\sin^2\lambda} \\
&= -\csc^2\lambda
\end{aligned}$$

$$\begin{aligned}
[10] \quad \frac{\cot^4\theta - \csc^2\theta}{\csc^2\theta} &= \frac{(\cot^2\theta - \csc^2\theta)(\cot^2\theta + \csc^2\theta)}{\csc^2\theta} \\
&= \frac{-(\csc^2\theta - 1 + \csc^2\theta)}{\csc^2\theta} \\
&= \frac{1 - 2\csc^2\theta}{\csc^2\theta} \\
&= \sin^2\theta - 2
\end{aligned}$$